

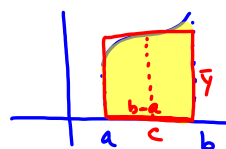
Review 21. properties of definite integrals
average value of a function

$$\int_a^b f(x) dx = - \int_b^a f(x) dx$$

$$\int_a^b k \cdot f(x) dx = k \int_a^b f(x) dx$$

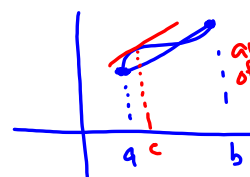
$$a < b < c \quad \int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$$

average value of a function $\bar{y} = \frac{1}{b-a} \int_a^b f(x) dx$



$$\bar{y}(b-a) = \int_a^b f(x) dx$$

$$\bar{y} = f(c) \quad \text{MVT for Integrals}$$



says that c exists

$$\frac{f(b) - f(a)}{b - a} = f'(c) \quad \text{MVT for derivatives}$$

Mar 21-8:56 AM

Mar 21-10:02 AM

Ex 1 $\int_2^5 f(x) dx = 12$ $\int_5^8 f(x) dx = 4$

a) $\int_2^8 f(x) dx = 16$

b) $\int_5^2 f(x) dx = -12$

c) $\int_2^6 f(x) dx + \int_6^8 f(x) dx = 16$

d) $\int_2^5 f(x) dx - \int_5^8 f(x) dx = 12 - 4 = 8$

$$\int_{-1}^7 f(x) dx = 14 \quad \int_4^{-1} f(x) dx = 3$$

$$\int_4^7 f(x) dx =$$

$$\begin{aligned} \int_{-1}^4 f(x) dx + \int_4^7 f(x) dx &= \int_{-1}^7 f(x) dx \\ \int_4^7 f(x) dx &= 14 - \int_{-1}^4 f(x) dx \\ &= 14 + \int_4^{-1} f(x) dx \\ &= 14 + 3 \\ &= 17 \end{aligned}$$

Mar 21-10:09 AM

Mar 21-10:13 AM

Ex 3 water flows into a tank at
a rate of $f(t) = t^2 \frac{\text{gal}}{\text{min}}$ what is
the average rate of water flow on $[0, 4]$
(ave value of the rate of flow)

$$\frac{1}{4-0} \int_0^4 t^2 dt = \frac{1}{4} \left. \frac{t^3}{3} \right|_0^4 = \frac{1}{4} \left(\frac{4^3 - 0^3}{3} \right)$$

not $\frac{4^2 - 0^2}{4-0} = 4$ ave rate of rate of flow $= \frac{16}{3}$

ave value of f' on $[a, b]$

$$\frac{1}{b-a} \int_a^b f'(x) dx = \frac{1}{b-a} (f(b) - f(a))$$

$$\text{ave rate of } f = \frac{f(b) - f(a)}{b-a}$$

Mar 21-10:17 AM

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